AN INTERMEDIATE HEATING AND COOLING METHOD FOR A DISTILLATION COLUMN

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To save energy in a distillation column, engineers have discussed the application of side-boilers and side-coolers and heat pumps to a distillation column. But no quantitative method which represents the relationship between temperature level and the minimum exchanging heat load of heat source and/or coolant at any plate has been developed. It is very difficult to design a distillation column with such equipment.

This paper represents the energy-saving effects of a side-boiler and a side-cooler on a distillation column on the basis of exergy concepts, and then proposes a quantitative method to determine the feasible domains for the use of a side-boiler and a side-cooler, and to clarify the important relationship mentioned above. This method can be used to design a distillation column with a heat pump and a multi-effective distillation system.

Introduction

Distillation is a unit operation which requires a large amount of energy. To decrease the energy consumption, engineers have tried to apply side-boilers, side-coolers[6,8,11,12] and heat pumps[3,7,10] to the distillation column. It might be effective for energy-saving to separate a binary mixture by using a heat-integrated multi-column system instead of a single column system[2,12,14]. Recently, a few distillation systems with such equipment for energy-saving have been realized.

There are different approaches to energy-saving in a distillation system, such as the following: 1) decrease of the summation of heating and/or cooling loads, 2) improvement of the temperature level of available heat sources or coolants, and 3) a combination of both 1) and 2).

This article describes an energy-saving design method for a distillation system which can effectively utilize several kinds of heat source and coolants around a distillation system. Firstly, a theoretical and quantitative consideration of energy-saving by side-boilers and side-coolers is discussed. Secondly, how the feasible domains for the use of side-boilers and side-coolers change according to the variations of feed conditions, specificcations of products, and vapor-liquid equilibrium is studied.

1. Energy Saving by Using a Side-boiler and a Side-cooler

This section will consider the advantages of adding side-boilers and side-coolers to a distillation system. The exergy concept based on the first and second laws of thermodynamics is a very powerful means of dealing with both the amount and the quality of heat energy.

Mass and heat balances are given over the conventional column model shown in **Fig. 1**. It is assumed that the temperature of the tops or the bottoms is equal to the boiling point of each mixture.

(Mass balance)

$$F=D+W \tag{1}$$

$$qFx_F+(1-q)FY_F=Dx_d+Wx_w \tag{2}$$

(Heat balance)

$$Q_r+qFc_F(T_F-T_o)+(1-q)F[c'_F(T_F-T_o)+\lambda_F]$$
$$=Q_c+Dc_d(T_c-T_o)+Wc_w(T_r-T_o) \tag{3}$$

where c_F and c'_F mean the specific heat of liquid mixtures with compositions x_F and y_F respectively.

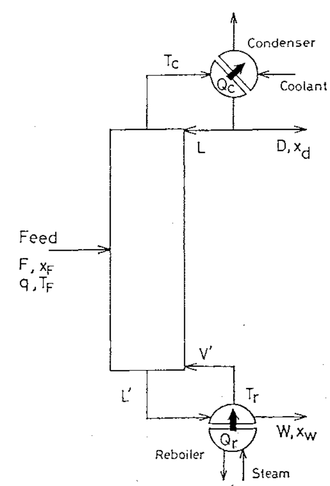

Fig. 1 Conventional distillation column model

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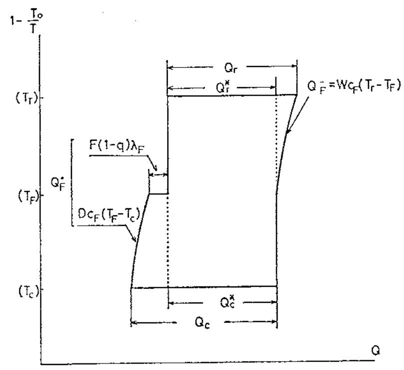


Fig. 2 $(1-T_0/T)-Q$ **diagram of a conventional distillation column**

The temperature of the surroundings is expressed by by T_o. These equations lead to

$$Q_r+Q_F^+=Q_c+Q_F^- \tag{4}$$

where

$$Q_F^+=D[\{qc_F+(1-q)c'_F\}(T_F-T_o)-c_d(T_c-T_o)]+(1-q)F\lambda_F \tag{5}$$

$$Q_F^-=W[c_w(T_r-T_o)-\{qc_F+(1-q)c'_F\}(T_F-T_o)] \tag{6}$$

Assuming $c_F=c'_F$, Eqs. (5) and (6) can be written as

$$Q_F^+=D[c_F(T_F-T_o)-c_d(T_c-T_o)]+(1-q)F\lambda_F \tag{7}$$

$$Q_F^-=W[c_w(T_r-T_o)-c_F(T_F-T_o)] \tag{8}$$

The $(1-T_o/T)-Q$ diagram for Eq. (4) under the condition of $c_F=c_w=c_d$ is shown in **Fig. 2**. The supplied heat energies are Q_r and Q_F^+ and the removed heat energies are Q_c and Q_F^-, in Eq. (4). Defining Q_r^* and Q_c^* as

$$Q_r^*=Q_r-Q_F^- \tag{9}$$

$$Q_c^*=Q_c-Q_F^+ \tag{10}$$

the following relation is obtained:

$$Q_r^*=Q_c^*=Q^* \tag{11}$$

Then, consider the exergy loss of a distillation column in the surroundings with the temperature, T_o, and the pressure, π, and of the pure nominal state for each component[8,9,13].

The exergies of a feed, e_F, the tops, e_d, and the bottoms, e_w are represented as follows:

$$e_F=F[c_F\{T_F-T_o-T_o\ln T_F/T_o\}+(1-q)(1-T_o/T_F)\lambda_F+RT_o\{x_F\ln x_F+(1-x_F)\ln(1-x_F)\}] \tag{12}$$

$$e_d=D[c_d\{T_c-T_o-T_o\ln T_c/T_o\}+RT_o\{x_d\ln x_d+(1-x_d)\ln(1-x_d)\}] \tag{13}$$

$$e_w=W[c_w\{T_r-T_o-T_o\ln T_r/T_o\}+RT_c\{x_w\ln x_w+(1-x_w)\ln(1-x_w)\}] \tag{14}$$

The values of e_F, e_d and e_w can be evaluated from the conditions of feed and products alone. The exergies of the supplied heat energy and the removed heat energy, e_r and e_c, are given as:

$$e_r=(1-T_o/T_r)Q_r \tag{15}$$

$$e_c=(1-T_o/T_c)Q_c \tag{16}$$

The exergy loss is calculated using Eqs. (12) to (16):

$$\Delta e_{\mathrm{conv}}=(e_r-e_c)+(e_F-e_d-e_w) \tag{17}$$

$$=T_o[(1/T_c-1/T_r)Q^*+(Q_F^+/T_c-Q_F^-/T_r)-(1-q)F\lambda_F/T_F]+\mathrm{const.} \tag{18}$$

$$\begin{aligned}\mathrm{const.}=&RT_o[F\{x_F\ln x_F+(1-x_F)\ln(1-x_F)\}\\&-D\{x_d\ln x_d+(1-x_d)\ln(1-x_d)\}\\&-W\{x_w\ln x_w+(1-x_w)\ln(1-x_w)\}]\\&-T_o[Fc_F\ln T_F/T_o-Dc_d\ln T_c/T_o\\&-Wc_w\ln T_r/T_o]\end{aligned} \tag{19}$$

The term of (e_r-e_c) in Eq. (17) is called "the net work consumption". Since $(e_F-e_d-e_w)$ can be evaluated only from the design conditions for a column, the exergy loss depends solely on the net work consumption. Moreover, the term "const." can be evaluated from the design conditions. Putting $\Delta e'_{\mathrm{conv}}=\Delta e_{\mathrm{conv}}-$ const., $\Delta e'_{\mathrm{conv}}$ is equal to the area around the heat availability lines in the $(1-T_o/T)-Q$ diagram[12) Fig. 2.

Now let us consider the exergy loss for a distillation column with a side-boiler. Denoting the temperature at a heat exchange plate by T_j and the heat energy supplied to a side-boiler by $\Delta Q_{r(j)}$, the exergies of the supplied and removed heat energies are given as:

$$e_r=(1-T_o/T_r)Q'_r \tag{20}$$

$$e_j=(1-T_o/T_j)\Delta Q_{r(j)} \tag{21}$$

$$e_c=(1-T_o/T_c)Q'_c \tag{22}$$

Where Q'_r means the heat energy transfered to a reboiler and Q'_c means the heat energy transfered from a condenser. The heat balance is given by:

$$Q'_r+\Delta Q_{r(j)}+Q_F^+=Q'_c+Q_F^- \tag{23}$$

and the exergy loss can be written as

$$\Delta e_{\mathrm{side}}=T_o[(1/T_c-1/T_r)Q'^*+(Q_F^+/T_c-Q_F^-/T_r)-(1-q)F\lambda_F/T_F+(1/T_r-1/T_j)\Delta Q_{r(j)}]+\mathrm{const.} \tag{24}$$

where

$$Q'^*=Q'_c-Q_F^+=Q'_c+\Delta Q_{r(j)}-Q_F^- \tag{25}$$

Consequently, the difference between Δe_{conv} and Δe_{side} becomes:

$$\Delta e_{\mathrm{conv}}-\Delta e_{\mathrm{side}}=T_o[(1/T_c-1/T_r)(Q^*-Q'^*)-(1/T_r-1/T_j)\Delta Q_{r(j)}] \tag{26}$$

As the condition of $Q^*=Q'^*$ is ordinarily satisfied, it becomes:

$$\Delta e_{\mathrm{conv}}>\Delta e_{\mathrm{side}} \tag{27}$$

Figure 3 shows $\Delta e'_{\mathrm{side}}(=\Delta e_{\mathrm{side}}-\mathrm{const.})$.

Similaly, it can be easily determined that the exergy loss of a distillation column with a sidecooler is smaller than that of a conventional column.

In general, the exergy loss of a distillation column with m side-boilers and n side-coolers is represented as follows:

$$\begin{aligned}\Delta e = T_0[&(1/T_c - 1/T_r)Q'^* + (Q_F^+/T_c - Q_F^-/T_r) \\ &-(1-q)F\lambda_F/T_F + \textstyle\sum_1^m (1/T_r - 1/T_j)\Delta Q_{r(j)} \\ &- \textstyle\sum_1^n (1/T_c - 1/T_k)\Delta Q_{c(k)}] + \text{const.}\end{aligned} \quad (28)$$

$$\begin{aligned}Q'^* &= Q'_c + \textstyle\sum_1^n \Delta Q_{c(k)} - Q_F^+ \\ &= Q'_r + \textstyle\sum_1^m \Delta Q_{r(j)} - Q_F^-\end{aligned} \quad (29)$$

2. The Relationship between Temperature and Exchanging Energy at the Plate with a Side-boiler or a Side-cooler

Supplying the very large heat energy, $\Delta Q_{r(j)}$, at the T_j temperature plate so that reboiler heat requirement, Q_r, becomes zero, is apparently impossible. In other words, there may be some relationship between T_j and the maximum value of $\Delta Q_{r(j)}$. The favorable conditions for the use of side-boilers and side-coolers are discussed in this section.

A conventional column is designed by using the information supplied by the minimum reflux, the minimum number of plates and so on. Assume the heat energies supplied to the reboiler and removed from the condenser under the minimum reflux condition as $Q_r^{\min}$ and $Q_c^{\min}$, respectively, and the component fraction and the temperature under the 'pinch' condition as x_p and T_p respectively. The operating line of the stripping section with the reboiler heat requirement, $Q_r(<Q_r^{\min})$ as shown in **Fig. 4**, is given as follows:

$$y_{j+1} = \frac{Q_r + W\{c'_{j+1}(T_{j+1}-T_0) - c_w(T_r - T_0) + \lambda_{j+1}\}}{Q_r - W\{c'_{j+1}(T_r - T_0) - c_j(T_j - T_0)\}} x_j - \frac{W\{c'_{j+1}(T_{j+1}-T_0) - c_j(T_j - T_0) + \lambda_{j+1}\}}{Q_c - W\{c_w(T_r - T_0) - c_j(T_j - T_0)\}} x_w \quad (30)$$

In general, $\Delta Q_{r(j)}$ supplied to the T_j temperature plate must satisfy the following condition in order to separate a mixture into the specified products:

$$\Delta Q_{r(j)} \geqq Q_r^{\min} - Q_r > 0 \quad (31)$$

where the equivalence is reached at minimum reflux. The difference between energy losses of the two cases to be supplied at the T_j plate and the $T_j+\Delta T$ plate on the operating line is obtained by:

$$\begin{aligned}&\Delta e_{\text{side}}(T_j) - \Delta e_{\text{side}}(T_j + \Delta T) \\ &= T_0\{1/(T_j+\Delta T) - 1/T_j\}\Delta Q_{r(j)}\end{aligned} \quad (32)$$

The exergy loss decreases as the heat exchanging plate temperature becomes lower (**Fig. 5**). On supplying the heat energy, $Q_r(<Q_r^{\min})$, to the reboiler, the lowest heat exchanging temperature, T_j^*, leading to the minimum exergy loss is equal to the temperature at the crossing point between the vapor-liquid equilibrium curve and the operating line given by Eq. (30). This point is called the pinch point in the case of a side-boiler (**Fig. 6**). In this case the number of plates is

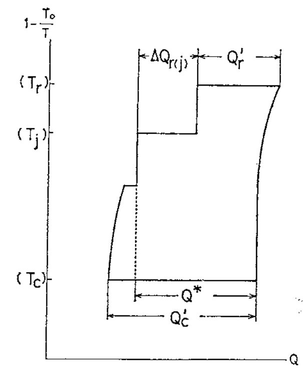


Fig. 3 $(1-T_0/T)$-Q **diagram of a distillation column with a side-boiler**

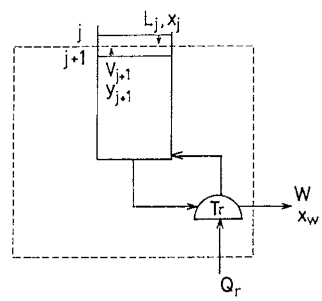


Fig. 4 Stripping section model

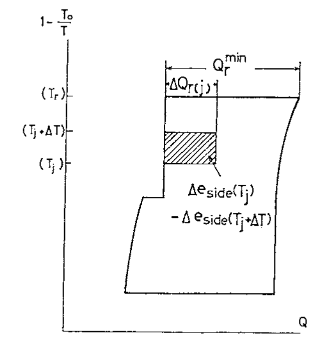


Fig. 5 Exergy loss due to plate temperature difference

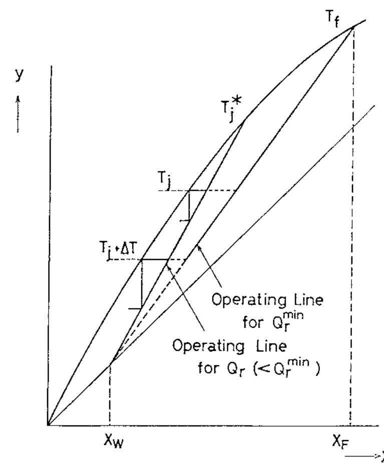
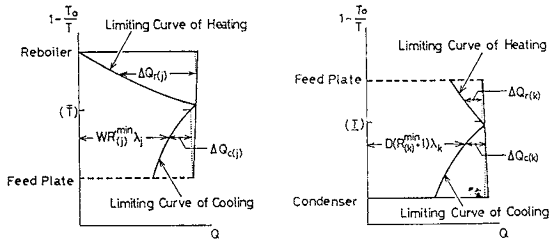


Fig. 6 x–y **diagram**



(a) $WR'^{\min}_{(j)}\lambda_j$ vs. T_j (stripping section)

(b) $D(R^{\min}_{(k)}+1)\lambda_k$ vs. T_k (enriching section)

Fig. 7

infinite. Consequently, under a given condition of arbitrary heat energy supplied to a reboiler or removed from a condenser, the temperature and the exchanging heat energy at the plate for minimum exergy loss can be determined by the pinch condition mentioned above.

If there is a pinch point at the k-th plate of the enriching section, the heat energy removed from a condenser, $Q^{\min}_{c(k)}$, can be derived from the following equations:

(Mass balance)

$$V_{k+1}=L_k+D \tag{33}$$

$$V_{k+1}y_{k+1}=L_kx_k+Dx_d \tag{34}$$

(Heat balance)

$$V_{k+1}[c'_{k+1}(T_{k+1}-T_\circ)+\lambda_{k+1}] \\ =L_kc_k(T_k-T_\circ)+Dc_d(T_c-T_\circ)+Q_c \tag{35}$$

(Pinch condition)

$$T_{k+1}=T_k,\ y_{k+1}=y_k,\ \lambda_{k+1}=\lambda_k \tag{36}$$

Therefore

$$Q^{\min}_{c(k)}=D(R^{\min}_{(k)}+1)\lambda_k+Q^{+}_{(k)} \tag{37}$$

where

$$R^{\min}_{(k)}=(x_d-y_k)/(y_k-x_k) \tag{38}$$

$$Q^{+}_{(k)}=D\{c_k(T_k-T_\circ)-c_d(T_c-T_\circ)\} \tag{39}$$

$D(R^{\min}_{(k)}+1)\lambda_k$ means the heat energy of the vapor at the k-th plate. Assuming the liquid and vapor mole fractions at the pinch point corresponding to the minimum reflux ratio are equal to x_p and y_p, the minimum total removed heat energy becomes:

$$Q^{\min}_{c}=Q^{\min}_{c(p)} \tag{40}$$

and the total supplied heat energy is given as follows:

$$Q^{\min}_{r}=Q^{\min}_{c}-Q^{+}_{F}+Q^{-}_{F} \tag{41}$$

On the other hand, if there is a pinch point at the j-th plate of the stripping section, the heat energy supplied to a reboiler, $Q^{\min}_{r(j)}$, is derived from the following equations:

(Mass balance)

$$L'_{j-1}=V'_{j-1}-W \tag{42}$$

$$L'_{j-1}x_{j-1}=V'_jy_j-Wx_w \tag{43}$$

(Heat balance)

$$Q_r+L'_{j-1}c_{j-1}(T_{j-1}-T_\circ) \\ =V'_j\{c'_j(T_j-T_\circ)+\lambda_j\}+Wc_w(T_r-T_\circ) \tag{44}$$

Therefore

$$Q^{\min}_{r(j)}=WR'^{\min}_{(j)}\lambda_j+Q^{-}_{(j)} \tag{45}$$

where

$$R'^{\min}_{(j)}=(x_j-x_w)/(y_j-x_j) \tag{46}$$

$$Q^{-}_{(j)}=W[c_w(T_r-T_\circ)-c_j(T_j-T_\circ)] \tag{47}$$

$WR'^{\min}_{(j)}\lambda_j$ means the heat energy of the vapor at the j-th plate. Assuming the mole fractions of the pinch point for the minimum reflux ratio are equal to x_p and y_p, the minimum total supplied heat energy is given as

$$Q^{\min}_{r}=Q^{\min}_{r(p)} \tag{48}$$

and in this case the summation of the removed heat energy can be calculated as follows:

$$Q^{\min}_{c}=Q^{\min}_{r}-Q^{-}_{F}+Q^{+}_{F} \tag{49}$$

Changing x_j or x_k in the regions $x_w<x_j\leqq x_f$ or $x_f\leqq x_k<x_d$, the relationships between the boiling point, T_j or T_k, and the heat duty at a reboiler or a condenser, $Q^{\min}_{r(j)}$ or $Q^{\min}_{c(k)}$, can be obtained. The section in the curve of $WR'^{\min}_{(j)}\lambda_j$ vs. T_j where $WR^{\min}_{(j)}\lambda_j$ increases with decreasing T_j is defined as the limiting curve of heating, and the section where $WR'^{\min}_{(j)}\lambda_j$ decreases is defined as the limiting curve of cooling. These lines are shown in **Fig. 7 (a)**.

Similarly, for the curve of $D(R^{\min}_{(k)}+1)\lambda_k$, the section where $D(R^{\min}_{(k)}+1)\lambda_k$ increases with decreasing T_k is defined as the limiting curve of heating and if $D(R^{\min}_{(k)}+1)\lambda_k$ decreases, the section is defined as the limiting curve of cooling. These lines are shown in **Fig. 7 (b)**.

Then, consider the maximum exchanging heat energy at the T_j temperature plate for a distillation system with a set of limiting curves of heating and cooling. (See example No. 4 in the case except for the system mentioned above.) Under the given value of the total heat energy requirements, $Q^{\min}_{r}$, $\Delta Q_{r(j)}$ supplied to the T_j plate on the limiting curve of heating can be easily obtained as follows: $\underline{T}$ denotes the limited temperature of using a side boiler. (See **Appendix**)

i) in the stripping section:

$$\Delta Q_{r(j)}=Q^{\min}_{r}-Q^{\min}_{r(j)} \tag{50}$$

ii) in the enriching section:

$$\Delta Q_{r(k)}=Q^{*\min}-D(R^{\min}_{(k)}+1)\lambda_k(T_F\geqq T_k>\underline{T}) \tag{51}$$

$\Delta Q_{r(j)}$ and $\Delta Q_{r(k)}$ are shown in Fig. 7 (a) and (b). Analogously, if the total removed heat energy, $Q^{\min}_{c}$, is fixed, the maximum heat energy removed from the T_j temperature plate on the limiting curve of cooling can be derived as follows: $\bar{T}$ denotes the limited temperature of using a side cooler. (See Appendix.)

i) in the stripping section:

$$\Delta Q_{c(j)}=Q^{*\min}-WR'^{\min}_{(j)}\lambda_j\ (\bar{T}>T_j\geqq T_F) \tag{52}$$

Table 1 Design conditions of examples

Example	1	2	3	4
	Benzene–Toluene		Ethanol–Acetic	
Feed rate F [kg-mol/hr]	50.0	50.0	50.0	50.0
Frac. x_F [mol%]	60.0	60.0	18.2	40.
q-value	1.0	0.0	1.0	1.0
Distillate x_d [mol%]	95.	95.	99.	99.0
Bottoms x_w [mol%]	5.0	5.0	1.0	1.0
Heat capacity (liquid)	Reference 13)			
Latent heat	Reference 13)			
V–L equi.	Reference 14)			

ii) in the enriching section:

$$\Delta Q_{c(k)} = Q_c^{*\min} - Q_{c(k)}^{\min} \tag{53}$$

$\Delta Q_{c(j)}$ and $\Delta Q_{c(k)}$ are shown in Fig. 7 (a) and (b).

(Example No. 1) Benzene-toluene distillation column

The feed conditions and the specifications of the products are given on **Table 1**. In this example the pinch point for the minimum reflux ratio is at the feed plate. The calculated values from Eqs. (4), (37) and (45) are shown in **Fig. 8**. (As the exergy loss will be not directly discussed below, $(1-T_0/T)$–Q diagram is substituted for T–Q diagram.) The location of Q^+ is different from that in Fig. 2 in order to present the maximum exchanging heat energies, $\Delta Q_{r(j)}$ and $\Delta Q_{c(k)}$, at the T_j and T_k temperature plates. The ab curve is the limiting curve of heating. The shadow area bounded by ab, $Q_r^{\min}$ and Q_F^- indicates the feasible domain to add side-boilers. On the other hand, the curve of bc is the limiting curve of cooling and it is possible to equip side-coolers in the shadow area bounded by ab, $Q_c^{\min}$ and Q_F^+.

(Example No. 2) Benzene-toluene distillation column

The design conditions of this example, except for $q=0$, are the same values as in example No. 1. The curves of abd and bc shown in **Fig. 9** agree with the curves of ab and bc for example No. 1.

(Example No. 3) Ethanol-acetic acid distillation column

The pinch point for the minimum reflux ratio is in the stripping section. The curves of ab and bcd shown in **Fig. 10** are the limiting curves of heating and cooling respectively. It is possible to add side-coolers not only in the enriching section but also in the part of the stripping section in order to recover high-temperature waste heat energies.

(Example No. 4) Ethanol-acetic acid distillation column

The limiting curves under the given design conditions are shown as ab, cd for heating and bc, df for cooling in **Fig. 11**. There is a switching point between cooling and heating operations in the stripping section. Since $WR'^{\min}\lambda_p$ under the minimum reflux ratio is larger than Q^*, the maximum removed heat energy in the stripping section is evaluated using Eq. (52). Each of the points a, b, c and d are located in the McCabe-Thiele diagram of **Fig. 12**. If heat energy more than $\Delta Q_{c(k)}$ is removed from the plate on the bc curve, it is necessary to supply heat energy in the cd section.

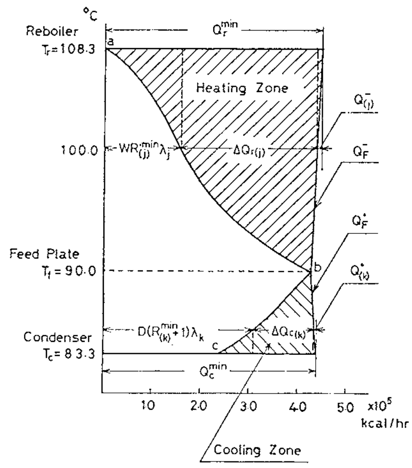


Fig. 8 T–Q diagram of Example No. 1

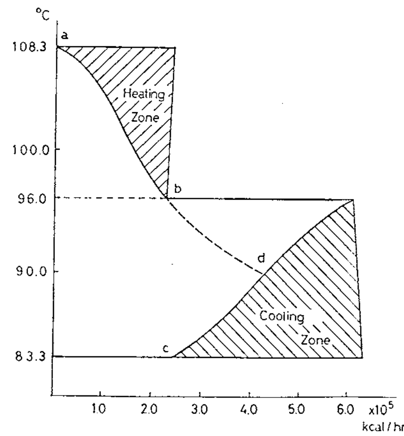


Fig. 9 T–Q diagram of Example No. 2

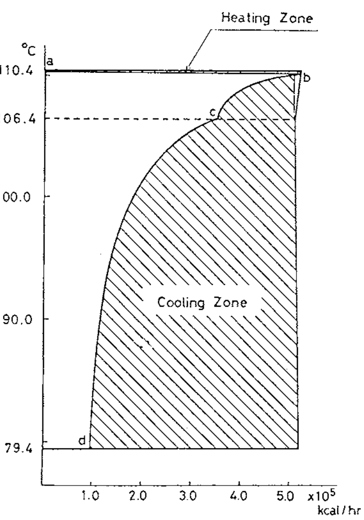


Fig. 10 T–Q diagram of Example No. 3

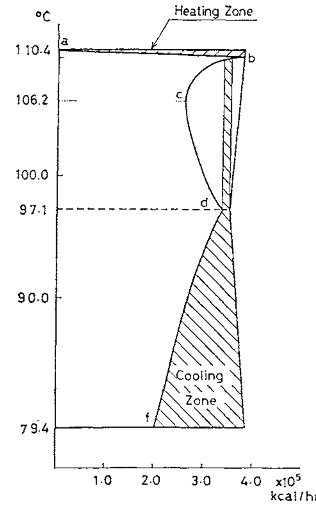


Fig. 11 ***T–Q* diagram of Example No. 4**

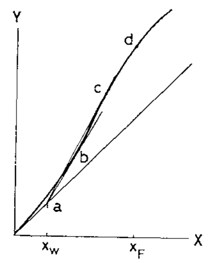


Fig. 12 ***x–y* diagram of Example No. 4**

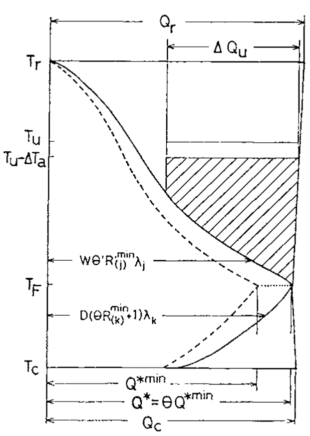


Fig. 13 ***T–Q* diagram of a distillation column with reflex ratio, $\theta R^{\min}$**

3. The Effective Arrangement of Side-Boilers and Side-Coolers

In general, it is assumed that the reflux ratio, R, is equal to $\theta R^{\min}$ ($\theta>1$) for the short-cut design method of a distillation column. If this assumption is satisfied in the design of a column with side-boilers and coolers, Eqs. (37) and (45) change to the following equations:

$$Q_{c(k)}=D(\theta R^{\min}_{(k)}+1)\lambda_k+Q^{+}_{(k)} \tag{54}$$

$$Q_{r(j)}=W\theta' R'^{\min}_{(j)}\lambda_j+Q^{-}_{(j)} \tag{55}$$

where

$$\theta'=\{D\theta R^{\min}_{(f)}+(1-q)F\}/WR'^{\min}_{(f)} \tag{56}$$

The total supplied and removed heat energies can be derived as follows.

(i) If the pinch point for the minimum reflux ratio is in the stripping section or at the feed plate,

$$Q_r=W\theta' R'^{\min}\lambda_p+Q^{+}_{(p)} \tag{57}$$

$$Q_c=Q_r-Q^{-}_F+Q^{+}_F \tag{58}$$

(ii) If the pinch point for the minimum reflux ratio is in the enriching section,

$$Q_c=D(\theta R^{\min}+1)\lambda_p+Q^{+}_{(p)} \tag{59}$$

$$Q_r=Q_c+Q^{-}_F-Q^{+}_F \tag{60}$$

Set $\theta=1.2$ for example No. 1, $Q_{c(k)}$ and $Q_{r(j)}$ are shown in **Fig. 13**.

When a utility with T_u temperature and Q_u heat energy is near a distillation column, the feasible domains of temperature and heat energy can be easily obtained in order to use it. If the utility satisfies the following condition, it is possible to add a side-boiler.

$$\bar{T}\geqq T_u-\Delta T_a\geqq T_j \tag{61}$$

where ΔT_a means the minimum approach temperature difference. Then the useful heat energy, ΔQ_u, is calculated as follows:

if $\Delta Q_{r(j)}\geqq Q_u$, $\Delta Q_u\leqq Q_u$

or if $\Delta Q_{r(j)}<Q_u$, $\Delta Q_u\leqq \Delta Q_{r(j)}$

Conclusion

In this paper, when there are several utilities around a distillation system, the method of using the energy sources is considered. By calculating the limiting curves of heating and cooling from the specifications of feed and products, vapor-liquid equilibrium and the pinch point conditions, it is easy to find the feasible domains for side-boilers and side-coolers and the relationship between the heat energy and the temperature of such a heat exchanger. Moreover, this method is a very powerful means of solving the design problems of a heat pump and the synthesis of a heat-integrated distillation system. The application of this method to these problems will be considered.

Appendix: The limiting temperature of using a side-boiler, $\underline{T}$ or a side-cooler, $\bar{T}$

At first, consider the limiting temperature of using a side-boiler. T_p means the temperature of the pinch point with minimum reflux, $T_c<T_p<T_F$. At the pinch point temperature, T_k, in $T_p<T_k<T_F$, the heat energy removed from a condenser, $Q^{\min}_{c(k)}$, is required using Eq. (33):

$$Q^{\min}_{c(k)}=D(R^{\min}_{(k)}+1)\lambda_k+Q^{+}_{(k)} \tag{A-1}$$

and the minimum total removed heat energy is given by Eq. (36):

$$Q^{\min}_{c(k)}=Q^{*\min}+Q^{+}_F \tag{A-2}$$

Under constant $Q^{\min}_c$, if the region with $Q^{\min}_c-Q^{\min}_{c(k)}>0$ is in $T_p<T_k<T_F$, operation using a side-boiler is feasible.

$$\Delta Q_{r(k)} = Q_o^{\min} - Q_c^{\min} \tag{A-3}$$

$$= Q^{*\min} - D(R_{(k)}^{\min}+1)\lambda_k + F(1-q)\lambda_F + D\{c_F(T_F - T_o) - c_k(T_k - T_o)\} \tag{A-4}$$

The third and the fourth terms in the foregoing equation mean heat sources except for side-boiler and a reboiler. Consequently, the heat energy supplied to a side-boiler at the j-th plate is

$$\Delta Q_{r(k)} = Q^{*\min} - D(R_{(k)}^{\min}+1)\lambda_k \tag{A-5}$$

The temperature to satisfy $\Delta Q_{r(k)}=0$ is defined as "the limiting temperature of using a side-boiler", $\underline{T}$. Though there is a limiting curve of cooling in $T_p < T_k < T_F$, it has to be neglected under the constant heat duty. Similarly, the limiting temperature of using a side-cooler, $\overline{T}$, has to be defined.

$$\Delta Q_{c(j)} = Q^{*\min} - WR'^{\min}_{(j)}\lambda_j = 0 \tag{A-6}$$

Nomenclature

c, c'	= specific heat at constant pressure	[kcal/kg-mol·ΔT]
D	= flow rate of the tops	[kg-mol/hr]
e	= exergy	[kcal/hr]
Δe	= exergy loss	[kcal/hr]
$\Delta e'$	= net work consumption	[kcal/hr]
F	= feed flow rate	[kg-mol/hr]
L	= internal reflux flow rate	[kg-mol/hr]
m	= number of side-boilers	
n	= number of side-coolers	
Q_c	= heat energy removed from a condenser	[kcal/hr]
Q_r	= heat energy supplied to a reboiler	[kcal/hr]
$Q_{c(k)}$	= heat energy removed from a condenser with pinch point at the k-th plate	[kcal/hr]
$Q_{r(j)}$	= heat energy supplied to a reboiler with pinch point at the j-th plate	[kcal/hr]
$\Delta Q_{c(k)}$	= heat energy removed from the k-th plate	[kcal/hr]
$\Delta Q_{r(j)}$	= heat energy supplied to the j-th plate	[kcal/hr]
Q_F^+, Q_F^-	= defined by Eqs. (5) and (6)	[kcal/hr]
$Q_{(k)}^+, Q_{(j)}^-$	= defined by Eqs. (39) and (47)	[kcal/hr]
Q^*, Q_c^*, Q_r^*	= defined by Eqs. (11), (10) and (9)	[kcal/hr]
q	= q-value	
R	= gas constant	[kcal·°C/kg-mol]
$R^{\min}, R'^{\min}$	= defined by Eqs. (38) and (46)	
T, T^*	= temperature	[°C] or [°K]
T_o	= surroundings temperature	[°C] or [°K]
$\overline{T}, \underline{T}$	= limiting temperature of using a side-cooler or side-boiler	[°C] or [°K]
ΔT_a	= minimum approach temperature difference	[°C]
V	= vapor flow rate	[kg-mol/hr]
W	= flow rate of the bottoms	[kg-mol/hr]
x	= liquid mole fraction of the light component	
y	= vapor mole fraction the light component	
π	= operating pressure	[mmHg]
λ	= latent heat	[kcal/ky-mol]
ω	= work supplied to a pump	[kcal/hr]

<Superscript>

min	= pinch point

<Subscripts>

c	= condenser or side-cooler
d	= the tops
F	= feed
f	= feed plate
j, k	= plate number
p	= the pinch point corresponding to minimum reflux ratio
r	= reboiler or side-boiler
u	= utility
w	= the bottoms

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