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EXTREME PERFORMANCE OF HEAT EXCHANGERS OF VARIOUS HYDRODYNAMIC MODELS OF FLOWS

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Abstract

The problem of minimization of entropy production is considered for one-pass heat exchangers of various types of description of hydrodynamic characteristics of the flows. Two models of the flows are considered, namely models of ideal mixing and ideal exclusion. The solution of the problem at issue allows one to construct a measure of thermodynamic perfectness of the heat exchanger taking into account the irreversibility of the heat exchange process.

Keywords: entropy production, heat exchanger.

1. Introduction

The irreversibility of a process of given average intensity can be used as a measure of the thermodynamic perfectness of this process. Entropy production σ takes the gauge of this irreversibility. So, for a heat exchange process the less σ the higher the temperature of a heated flow, other things being equal. It means that the energetic value of the heated flow decreases with respect to σ . If one can control the parameters of one of the flows at each section of the heat exchanger then the minimal value of entropy production σ_{\min} can be reached. The problem of σ_{\min} determination is solved in [1, 2].

But it is impossible to control the process inside the apparatus. Practically the parameters of the flows can be changed at the entrance of the apparatus only. These parameters are temperature and flow velocity. Inside the heat exchanger these parameters change according to the structural design of the apparatus. In β] this problem was considered and it was proved that the minimum of entropy production corresponds to counterflow heat exchangers.

The counterflow heat exchanger has already been investigated thoroughly. For example, heat exchange process and pressure drop contributions to the irreversibility were shown in [4]. [5] introduces the dependence of viscosity on temperature and shows the resulting effects on entropy production. But real schemes of heat

exchangers can differ from the counterflow one significantly. That is why other models of heat exchangers should be considered too.

Let us consider one-pass heat exchangers (recuperators) and assume the flow velocities to be constant inside the heat exchanger. Two types of hydrodynamic models of the flows are considered. They are models of ideal mixing and ideal exclusion [6]. Four types of heat exchangers can be obtained by combining these two models. They are shown in *Fig. 1*. It is obvious that the entropy production for each of these types cannot be less than σ^{\min} . But there exists a lower boundary of entropy production for each type of heat exchangers. Further these boundaries will be found.



Fig. 1. Structures of heat exchangers described by different hydrodynamic models of the flows: (*a*) 'mixing-mixing', (*b*) 'exclusion-mixing', (*c*) 'mixing-exclusion', (*d*) 'exclusion-exclusion'.

2. The Problem Formalization

Let us consider a heat exchanger (*Fig. 2a*) consisting of two chambers such that there exists counterflow there. Heat flux at each cross section of the apparatus depends on parameters of the flows at this section. Parameters of one of these flows are given. Let us call this flow 'the fixed flow'. Parameters of another flow ("controlled flow") should be chosen at the inlet of the heat exchanger (l = 0) to minimize entropy production. Let the intensity of heat exchange be given by linear (Newtonian) law [7]. The total intensity of the heat flow is given as :

$$\int_{0}^{L} q(T_1, T_2) dl = \int_{0}^{L} \alpha \Big(T_1(l) - T_2(l) \Big) dl = q_0, \tag{1}$$

where α is specific (related to the unit of length) coefficient of heat transfer, T_1 , T_2 are temperatures of fixed and controlled flows averaged with respect to the area of the corresponding chamber section, respectively.



Fig. 2. Schematic of the flows in a heat exchanger. (a) counterflow, (b) one-direction flow

If the fixed flow is described by the model of ideal exclusion then its entropy change rate is determined as follows:

$$s_1 = w_1 \ln \frac{T_{1e}}{T_{1i}} = w_1 \ln \frac{T_{1i}w_1 - q_0}{T_{1i}w_1},$$
(2)

where w_1 is time rate heat capacity (product of heat capacity and flow rate) of the fixed flow, subscripts *i* and *e* indicate *inlet* and *exit* of the apparatus, respectively. If the fixed flow is described by the model of ideal mixing then the temperature in the chamber is constant and equals its temperature at the exit of the heat exchanger. Therefore

$$s_1 = -\frac{q_0}{T_{1e}} = \frac{w_1 q_0}{q_0 - w_1 T_{1i}}.$$
(3)

Note that the rate of entropy change of the fixed flow is determined and does not depend on control variables (temperature T_{2i} and time rate heat capacity of the controlled flow w_2). That is why the problem of extreme performance of heat exchangers can be formalized as follows:

$$s_2 = \int_0^L \frac{q(T_1, T_2)}{T_2(l)} dl \to \min_{T_{2i}, w_2}$$
(4)

subject to (1) and

$$\frac{dT_{\nu}}{dl} = \frac{q(T_1, T_2)}{w_{\nu}}, \qquad \nu \in \{1, 2\},$$
(5)

where $T_i(0)$ is fixed, if the *i*-th flow is described by the model of ideal exclusion.

3. Structure 'Exclusion-Exclusion'

It is proved in [2] that the ratio of the agents' temperatures T_2/T_1 should be constant to minimize entropy production in the heat exchanger:

$$\frac{T_2}{T_1} = k = 1 - \frac{w_1}{\alpha L} \ln \frac{T_{1i}}{T_{1e}}.$$
(6)

On the other hand it follows from (5) that this ratio is inversely proportional to the ratio of water equivalents of the flows:

$$w_2 = \frac{w_1}{k}.\tag{7}$$

So, for the section l = 0

$$T_{2i} = kT_{1e} = k\left(T_{1i} - \frac{q_0}{w_1}\right).$$
(8)

Thus the vector of controls (T_{2i}, w_2) is obtained. These values allow one to maintain the optimal temperature profile inside the heat exchanger. The minimal rate of increase of controlled flow entropy is equal to

$$s_2^* = w_2 \ln \frac{T_{2e}}{T_{2i}} = \frac{w_1}{k} \ln \frac{T_{1i}w_1}{T_{1i}w_1 - q_0}.$$
(9)

4. Structures 'Mixing-Mixing' and 'Mixing-Exclusion'

If the fixed flow is described by the model of ideal mixing then its temperature inside the apparatus is constant. To maintain the optimal regime (constant ratio of the flows' temperatures) the temperature of the controlled flow should be constant too. It corresponds to ideal mixing of the controlled flow or infinite velocity of the controlled flow if it is described by the model of ideal exclusion. Here temperature T_2 can be determined from (1):

$$T_2 = T_{1e} - \frac{q_0}{\alpha L} \tag{10}$$

and the rate of the controlled flow entropy increase is

$$s_2^* = \frac{q_0}{T_{1i} - q_0 \left(\frac{1}{w_1} + \frac{1}{\alpha L}\right)}.$$
(11)

Infinite rate of the controlled flow cannot be practically reached. Therefore it should be maintained at the upper boundary corresponding to u_2^{max} . In this case temperature T_{2i} should be chosen to fulfill condition (1)

$$T_{2i} = T_{1i} - q_0 \left[\frac{1}{\omega} + \frac{1}{w_1} \right],$$
 (12)

where

$$\omega = w_2^{\max} \left[1 - \exp\left(-\frac{\alpha L}{w_2^{\max}}\right) \right]$$
(13)

and the rate of the controlled flow entropy increase is

$$s_{2}^{*} = w_{2}^{\max} \ln \left[1 + \frac{q_{0}}{w_{2}^{\max} \left[T_{1i} - q_{0} \left(\frac{1}{\omega} + \frac{1}{w_{1}} \right) \right]} \right].$$
 (14)

The dependency of $\tilde{s}_2 = s_2^*/(\alpha L)$ with respect to $\tilde{w}_2^{\text{max}} = w_2^{\text{max}}/(\alpha L)$ (both these expressions are dimensionless) is shown in *Fig. 3a*, and plots of the same variable \tilde{s}_2 with respect to $\tilde{q} = q_0/(\alpha L T_{\text{li}})$ are shown in *Fig. 3b* for different values of \tilde{w}_2^{max} .

5. Structure 'Exclusion-Mixing'

Here the fixed flow is described by the model of ideal exclusion. The rate of entropy increase for the controlled flow is

$$s_2 = \frac{q_0}{T_{2e}} = \frac{w_2 q_0}{T_{2i} w_2 + q_0}.$$
 (15)

To determine the vector of controls (T_{2i}, w_2) let us solve Eq. (5) for the fixed flow taking into account that the temperature of the controlled flow inside the chamber is constant:

$$\frac{dT_1}{dl} = \frac{\alpha}{w_1} \Big(T_1(l) - T_{2e} \Big), \quad T_1(0) = T_{1e}.$$
(16)

The solution of this equation is

$$T_1(l) = T_{2e} + \left(T_{1e} - T_{2e}\right) \exp \frac{\alpha l}{w_1}.$$
 (17)

Substituting (17) into (1) one can express w_2 as follows:

$$\frac{1}{w_2} = \frac{1}{q_0} \left(T_{1e} - T_{2i} \right) - \frac{1}{w_1 \left[\exp \frac{\alpha L}{w_1} - 1 \right]}.$$
(18)





Fig. 3. Dependencies of entropy increment rate of the controlled flow \tilde{s}_2 with respect to (*a*) time rate heat capacity \tilde{w}_2 ($\tilde{q} = 0.5$, $\tilde{\alpha} = (\alpha L)/w_1 = 0.01$, \tilde{s}_2^* corresponds to the structure 'mixing-mixing') and (*b*) heat transfer flux \tilde{q} ($\tilde{\alpha} = 0.1$): (1) $\tilde{w}_2 = 0.25$, (2) $\tilde{w}_2 = 0.5$, (3) $\tilde{w}_2 = 1.0$, (4) $\tilde{w}_2 = 2.0$, (5) $\tilde{w}_2 \to \infty$.

To find the rate of entropy change for controlled flow let us substitute the found controls into (15):

$$s_2^* = \frac{q_0}{T_{1i} - q_0 \left[\frac{1}{w_1} + \frac{1}{w_1 \left(\exp\frac{\alpha L}{w_1} - 1\right)}\right]}.$$
(19)

6. One-Direction Flow System of the 'Exclusion-Exclusion' Structure

Let us consider the last possible structure of the one-pass heat exchangers namely a one-direction flow system (*Fig. 2b*). The differences are possible only for the 'exclusion–exclusion' structure here. To find temperatures of the agents one should

solve the following system of differential equations (it follows from (5)):

$$\frac{dT_1}{dl} = -\frac{\alpha}{w_1} (T_1 - T_2), \qquad T_1(0) = T_{1i},
\frac{dT_2}{dl} = \frac{\alpha}{w_2} (T_1 - T_2), \qquad T_2(0) = T_{2i}.$$
(20)

The solution of system (20) is:

$$T_{1}(l) = T_{1i} - \frac{w_{2}}{w_{1} + w_{2}} (T_{1i} - T_{2i}) \left[1 - \exp\left(-\alpha \frac{w_{1} + w_{2}}{w_{1} w_{2}}l\right) \right],$$

$$T_{2}(l) = T_{2i} + \frac{w_{1}}{w_{1} + w_{2}} (T_{1i} - T_{2i}) \left[1 - \exp\left(-\alpha \frac{w_{1} + w_{2}}{w_{1} w_{2}}l\right) \right].$$
(21)

Problem (4), (1) taking into account (21) which allows one to omit conditions (5) has the form:

$$s_2 = w_2 \ln \frac{T_{2e}(T_{2i}, w_2)}{T_{2i}} \to \min_{T_{2i}, w_2}$$
 (22)

subject to

$$\frac{w_1 w_2}{w_1 + w_2} \left(T_{1i} - T_{2i} \right) \left[1 - \exp\left(-\alpha L \frac{w_1 + w_2}{w_1 w_2} \right) \right] = q_0.$$
(23)

Expressing T_{2i} from (23) as a function of w_1 , q_0 and substituting it into (22) one can transform this problem to the following problem on unconditional minimization:

$$\tilde{s}_2 = \tilde{w}_2 \ln \left[1 + \frac{\tilde{q}}{\tilde{w}_2 [1 - \tilde{q} \xi(\tilde{w}_2)]} \right] \to \min_{\tilde{w}_2}, \tag{24}$$

where $\tilde{s}_2 = \frac{s_2}{\alpha L}$, $\tilde{w}_2 = \frac{w_2}{\alpha L}$, $\tilde{q} = \frac{q_0}{\alpha L T_{\text{li}}}$ and

$$\xi = \frac{w_1 + w_2}{w_1 w_2} \frac{\alpha L}{1 - \exp\left(-\alpha L \frac{w_1 + w_2}{w_1 w_2}\right)}$$
(25)

are dimensionless variables. Function $\tilde{s}_2(\tilde{w}_2)$ is a monotonously decreasing function at the set of physically possible values of \tilde{w}_2 . That is why the solution of this problem is $\tilde{w}_2 \to \infty$. It means that the rate of the controlled flow should be infinitely large. In such a case the considered structure coincides with the structure 'exclusionmixing'. If \tilde{w}_2 is restricted by upper boundary \tilde{w}_2^{max} then the value of the controlled flow entropy change rate s_2^* and corresponding temperature T_{1i} are calculated as follows:

$$s_{2}^{*} = w_{2}^{\max} \ln \left[1 + \frac{\alpha L q_{0}}{w_{2}^{\max} [\alpha L T_{1i} - q_{0} \xi(w_{2}^{\max})]} \right],$$

$$T_{2i} = T_{1i} - \frac{q_{0} \xi(w_{2}^{\max})}{\alpha L}.$$
(26)

Qualitatively, the plots $\tilde{s}_2(\tilde{w}_2^{\max})$ and $\tilde{s}_2(\tilde{q})$ are the same as the ones for the structure 'mixing–exclusion' shown in *Fig. 3*.

7. Comparison of Various Types of Recuperators

To compare heat exchangers of various types, one needs to calculate entropy production σ^* for each type of apparatus at the same value of heat flow q_0

$$\sigma^*(q_0) = s_1(q_0) + s_2^*(q_0), \tag{27}$$

where s_1, s_2^* are rates of entropy change for both flows: s_1 is calculated using either (2) or (3); s_2^* is found in previous subsections (9), (11) or (19).

Let us use the following dimensionless parameters:

$$\tilde{\alpha} = \frac{\alpha L}{w_1}, \quad \tilde{q} = \frac{q_0}{\alpha L T_{1i}}, \quad \tilde{\sigma} = \frac{\sigma^*}{\alpha L}.$$

Dependencies $\tilde{\sigma}(\tilde{q})$ are represented in *Table 1* for all types of the considered heat exchangers and plots of these dependencies are depicted in *Fig. 4*. It should be noted that dependencies $\tilde{\sigma}(\tilde{q})$ are boundaries of permissible values of vector $(\tilde{\sigma}, \tilde{q})$ for real heat exchanger of the same type. The closer the real value of this vector to its boundary, the higher the thermodynamic perfectness of the heat exchanger.

8. Technical Application: Heat Recovery in Ventilation Systems

New building standards such as low energy house or solar passive house set high requirements to the heat recuperator. The purpose is to minimize the ratio of energy expenditures for operation and amount of heat transferred from one flow to another one. For instance, the German Institute for Passive House Building (Passivhaus Institut) recommends systems with a heat return factor of 75% at least [8].

Let us first of all explain the difference between standard performance ratio and the proposed approach. The theoretical recovery factor obtained from adiabatic measurement (i.e. no heat losses through the recuperator envelope) is given in respect to the notation of *Fig.* 5, by:

$$\eta = \frac{T_{\rm li} - T_{\rm le}}{T_{\rm li} - T_{\rm 2i}}.$$
(28)

Table 1.	Dependencies of the minimal production of entropy with respect to heat flow for	•
	different types of heat exchangers	

Controlled flow		Fixed flow (hot agent)			
(col	d agent)	Ideal exclusion	Ideal mixing		
Ideal mixing		$\frac{\tilde{q}\left(1-e^{\tilde{\alpha}}\right)}{1-e^{\tilde{\alpha}}\left(1-\tilde{\alpha}\tilde{q}\right)}+\frac{\ln\left(1-\tilde{\alpha}\tilde{q}\right)}{\tilde{\alpha}}$			
	one-direction		$\frac{\tilde{q}}{1-\tilde{a}(1+\tilde{\alpha})}-\frac{\tilde{q}}{1-\tilde{\alpha}\tilde{a}}$		
Ideal	flow				
exclusion	counterflow	$\frac{1}{\tilde{\alpha}} \frac{\ln^2 \left(1 - \tilde{\alpha} \tilde{q}\right)}{\tilde{\alpha} + \ln \left(1 - \tilde{\alpha} \tilde{q}\right)}$			



Fig. 4. Dependencies of minimal entropy production $\tilde{\sigma}$ with respect to averaged intensity of the heat exchange process \tilde{q} for the following structures: (1) 'mixing-mixing' and 'mixing-exclusion', (2) 'exclusion-mixing' and one-directional flow recuperators, (3) 'exclusion-exclusion'.

This factor takes into account temperature only, therefore it is useful when no water condenses from the outgoing stream (hot side). As a result of this restriction mentioned above, there were introduced some other indexes [9, 10] based on changes of enthalpy of the flows. All of these indexes compare the real efficiency of the heat exchanger and the reversible boundary. But the process of heat exchange is irreversible. The proposed method stems from the idea of designing a new comparison

method not based on ideal behaviour but on reality. This leads to the use of entropy production to evaluate the thermodynamic perfectness of the apparatus. The distance between the point at the space $(\tilde{q}, \tilde{\sigma})$ corresponding to the operation regime of the heat exchanger and the boundary line represents the operation perfectness of the device with respect to what is physically possible considering given technical features. This method offers an easy way for a customer to compare the effectiveness and the quality of design of two models competing, or for a manufacturer to improve its product.



Fig. 5. Schematic representation of a compact ventilation system unit for single or multiple dwelling airing

In the example below, we intend to show how entropy production rate enables the comparison of recovery performance of two different models of counterflow air to air heat exchangers.

The recovery system includes the heat exchanger itself, air inlet filter and ventilators (*Fig.* 5). The obtained results do not take into account the heat produced by ventilators, increasing the temperature of the streaming air on both sides. That is why the data received from the manufacturer (temperatures at the input and output of the system) should be recalculated to get the temperatures at the inlet and outlet of the heat exchanger. In this respect, we eliminate in the example below, the heat energy released by the two fans by calculating two corrected temperatures T_{2i} from the temperature T_{1v} and T_{2v} given by the manufacturer and from the power consumption of the two ventilators (*Table* 2).

With the set of data T_{1i} , T_{1e} , T_{2i} , and T_{1i} , we can easily calculate the entropy production of the two fluxes because in this case heat transfer rate is dominant compared to the entropy production due to viscosity [5]:

$$s_{\nu} = V_{\nu} \rho C_p \ln \frac{T_{\nu e}}{T_{\nu i}}, \quad \nu \in \{1, 2\},$$
(29)

where V_{ν} is the volume rate of the ν -th flow, ρ is the density of the air, C_p is the heat capacity of the air.

In order to evaluate the perfectness of the apparatus, we first draw up the curve of minimal entropy production considering the technical features of the investigated device. In a second step, we mark in the plot the three operation points (*Figs.*6, 7).

			Operation point			
	Sensor		1	2	3	
Controlled	Outside air	T_{2i} ,	°C	-3.0	4.0	10.0
temperature	Extract air	T_{1v} ,	°C	21.0	21.0	21.0
Measured	Supply air	T_{2v} ,	°C	20.1	20.6	20.8
temperature	Exhaust air	T_{1e} ,	°C	4.4	9.3	13.3
Corrected	Recuperator outlet	T_{2e} ,	°C	19.9	20.4	20.7
temperature	Recuperator inlet	T_{1i} ,	°C	21.1	21.2	21.1

Table 2. Determination of operating values



Fig. 6. Results for a single dwelling ventilation device: 1. curve of minimal entropy production, 2. depicts operation points with flow rate of 100 m³/h, 3. depicts operation points with flow rate of 125 m³/h

The plots depicted in *Figs.* 6 and 7 reveal that the higher the flow rate, the lower the



Fig. 7. Results for a multiple dwelling ventilation device: 1. curve of minimal entropy production, 2. depicts operation points with flow rate of $200 \text{ m}^3/\text{h}$, 3. depicts operation points with flow rate of $270 \text{ m}^3/\text{h}$

corresponding entropy production. However this calculation exclusively focuses on heat exchange within the heat recuperator and neglects entropy increase in the ventilators, where sources of entropy production are pressure as well as temperature rise.

In the next article, we will expand on the calculation of entropy in recovery systems, including ventilator contribution and the internal and external air leaks occurring in such apparatus, dictated by their design.

9. Conclusion

The problem of extreme performance of one-pass heat exchangers of different hydrodynamic models of the flows is considered. The obtained results allow one to construct a criterion of thermodynamic perfection of heat exchangers taking the extreme performance boundary as an ideal regime to compare with. Such a criterion takes into account unremovable losses namely losses due to irreversibility (as it had been done in [1] and [5]) and due to hydrodynamics of the flows. This criterion can be used, for example for performance comparison of heat exchangers of either the same size features or the same hydrodynamic characteristics of the flows.

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List of Symbols

C_n	heat	capad	city	of	the	air
- 0						

- k^{ν} a constant
- *l* length coordinate
- *L* length of the apparatus
- *q* specific heat flux
- q_0 total heat flux
- s_1, s_2 entropy increment rate of the flows
- T_1, T_2 temperatures of the flows
- V_1, V_2 volume rates of the flows
- w_1, w_2 product of heat capacity and flow rate

Greek Letters

- α heat conductance coefficient
- η recovery factor
- ν enumerate variable
- ξ, ω additional variables defined by (25), (13), respectively
- ρ density of the air
- σ entropy production

Subscripts and Superscripts

- dimensionless variable
 optimal value
 max maximal value
 min minimal value
 at the exit point
- i at the inlet point
- v indoor port of the system

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