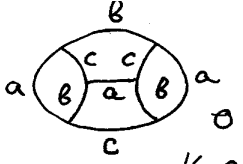
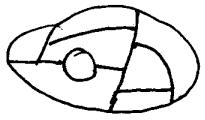



$a, b, c$ – эл-ты $\mathbb{Z}_2 \oplus \mathbb{Z}_2$ группы Клейна
три пересеч. границы прибавляем ребро.

$\forall G_1, G_2 \in \Gamma$, $G_1 * G_2 = \theta * G_1$, где $\theta = \Theta$

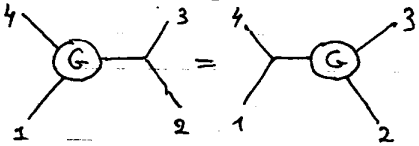


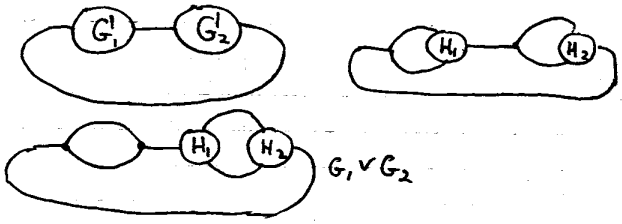
$\Lambda$-модули Вожеля
Vogel

$V : \Gamma \times \Gamma \to \Gamma$ вершинное умнож.

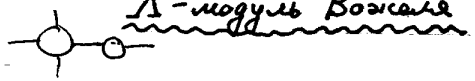
**Теорема:** $G_1 * G_2 = \theta * (G_1 \vee G_2)$

**Лемма**

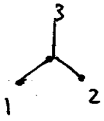


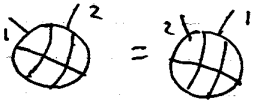

$\Lambda$-модуль Вожеля

$B$ 

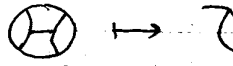
$X = \{$ графы с 1, 3-вал. верш., 1-вал. верш., пронумер. $\}$ / AS, IHX

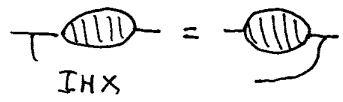
 $\neq 0$ $\quad X_2$ $\forall x \in X_2 \quad \sigma(x) = x$

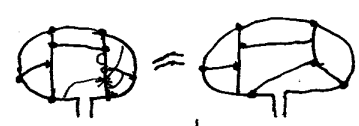
$\sigma$ – перест. ног

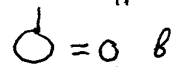
 $X_2 \cong B_2$

$\Gamma \to X_2$ разрезание ребра

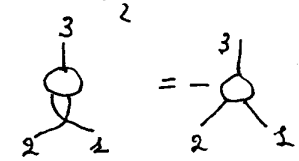


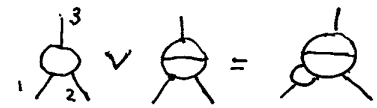


 $= 0$ в $\Gamma$

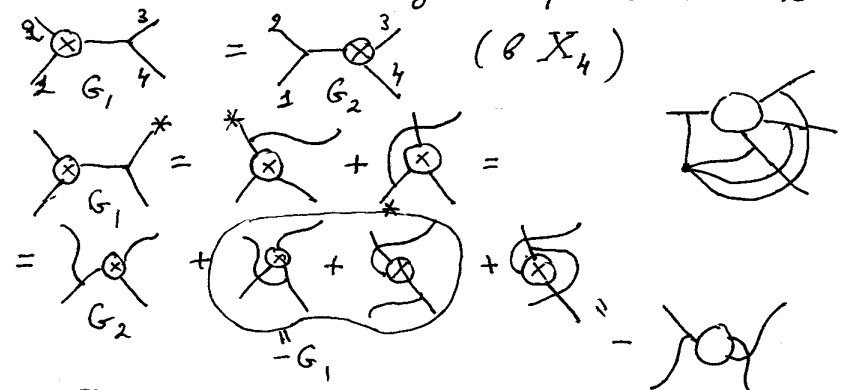
$\Lambda \subset X_3$ антисимм. отн. $S_3$
алгебра Вожеля



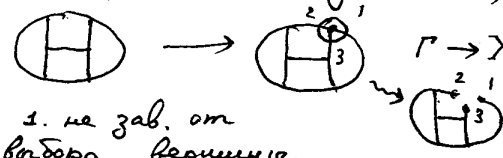


**Лемма Вожеля – Дасбаха**

Вставка $x \in \Lambda$ в соседние вершины элемента $X$




$G \in \Gamma$ выбросим вершину $v \in V(\Gamma)$



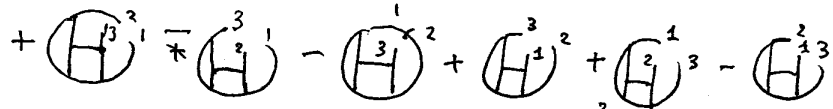
$\Gamma \to X_3$

**Факты.** 1. не зав. от выбора вершины

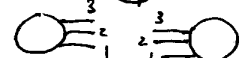
2. результат симметричен отн. $A_3 \subset S_3$

**Гипотеза** $\Gamma \to \Lambda$.

$\frac{1}{6} \sum_{\sigma \in S_3} \operatorname{sgn}(\sigma)\, G_v^{\sigma}$ $\quad v$ удал. $\sigma$ перест. ноги



$G_1$ и $G_2$ $\quad G_1, G_2 \in \Gamma$

 $G_1 \vee G_2$