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COMBINATORICS

Lecture 10: Composition of virtual species

In the previous lecture we defined the ring of virtual species equipped with the operations of addition and multiplication. You have seen that the definitions of these operations are straighforward and present no difficulty. In contrast with this, the (partitional) *composition* of virtual species is a highly non-trivial operation. To be more exact, it is easy to define the composition of a difference of two species of structures on the *left* by setting $(F-G)\circ H = (F\circ H)-(G\circ H)$, but it is not easy to give a consistent definition for the composition $F\circ(H-K)$. To do this, we will need the notion of *multisort species*.

Definition. A *multiset* is a finite set U given together with a splitting of U into a number of disjoint subsets $U = U_1 \cup ... \cup U_k$ (some of the subsets U_i may be empty). Elements of U_i are referred to as *points of sort* i. In other words, the structure of multiset is a function $\chi : U \to [k]$ with $U_i = \chi^{-1}(i)$ for each $i = 1, 2, ..., k$.

Definition. A species of structures on k sorts is a functor from the category $\mathbb{M}_k$ of k-multisets and k-bijections into the category $\mathbb{S}$ of finite sets and maps.

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